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A holographic view on matrix model of black hole

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Abstract: We investigate a deformed matrix model proposed by Kazakov et.al. in relation to Witten's two-dimensional black hole. The existing conjectures assert the equivalence of the two by mapping each to a deformed $c = 1$ theory called the sine-Liouville theory. We point out that the matrix theory in question may be naturally interpreted as a gauged quantum mechanics deformed by insertion of an exponentiated Wilson loop operator, which gives us more direct and holographic map between the two sides. The matrix model in the usual scaling limit must correspond to the bosonic $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ theory in genus expansion but exact in α'. We successfully test this by computing the Wilson loop expectation value and comparing it against the bulk computation. For the latter, we employ the α'-exact geometry proposed by Dijkgraaf, Verlinde, and Verlinde, which was further advocated by Tseytlin. We close with comments on open problems.

Keywords: Black Holes in String Theory, M(atrix) Theories, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

Recently, the matrix model of two dimensional noncritical string theory came under spotlight in a new context. When the matrix model was incepted some fifteen years ago,[1] it started as a theory of triangulated worldsheet whose structure is encoded in the Feynman diagram of the quantum mechanics in question. Under the new interpretation, however, the matrix is more than such an auxiliary field. It is reinterpreted as open string tachyons living on many unstable D-branes, whose condensation leads to the two dimensional string theory [2]. In this new interpretation, restriction to the singlet sector becomes quite natural since the open string side typically has gauge symmetry.

On the other hand, it is well-known that the two-dimensional string theories admit black hole backgrounds [3, 4, 5]. This background was first considered by Witten, as a coset model of $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ type. Relationship among this theory, usual $c = 1$ noncritical string theories, and matrix models has been the subject of a good deal of interests since early 1990's. For the case with $\mathcal{N}$ =(2,2) supersymmetries on the worldsheet, there is a strong evidence that the black hole theory is in fact the same as the super-Liouville theory [6]. For cases with lesser supersymmetries, however, the situation is slightly different. For instance, in the bosonic case, the black hole theory is conjectured by Fateev, Zamolodchikov, and Zamolodchikov (FZZ) [7] to be reached by perturbing usual Liouville theory with condensation of vortices, leading to so-called sine-Liouville theory.

Recently, a matrix model is proposed by Kazakov, Kostov, and Kutasov in [8] to be dual to this sine-Liouville theory, which would then also imply that the matrix model will describe the black hole theory upon using the FZZ conjecture. This matrix model can be regarded as a deformation of the $c = 1$ matrix model. Their calculation of the matrix

[1] See ref. [1] for a general review of the original matrix model proposal.

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model supports the equivalence between the matrix model and the sine-Liouville theory. Although the proposal is convincing, a more direct relation between the matrix model and the black hole would be desirable.

In this paper we employ a holographic view point on this matrix model, and show a rather direct map to the black hole. Our demonstration is based on the observation that the matrix model in [8] is actually a gauge theory. Then, in analogy with the well-known AdS/CFT correspondence [9], it is natural to assume that the expectation value of Wilson loop operator is calculated from a partition function of a string whose boundary is fixed on a circle on which the Wilson loop is defined. Thus, from this observable one can extract some geometric data for the target spacetime of the holographically-dual string theory. We will show that the result obtained from the gauge theory side nicely fits with the one obtained from the α'-exact string background [10, 11].

This paper is organized as follows. Section 2 contains a brief review on the matrix model. In section 3, we explain that the matrix model is actually a gauge theory, then discuss the holographic description of the matrix model. In section 4 we compute the Wilson line expectation value in the matrix side and compare it to the expected results from the proposed closed string dual, namely the two dimensional black hole. Section 5 is devoted to discussion.

2. Review of a matrix model of two-dimensional black hole

In this section, we briefly review the proposal of [8] on a matrix model which describes a string theory defined on a two-dimensional black hole. See also [12] for a comprehensive review.

Consider the $c = 1$ matrix quantum mechanics compactified on a circle with radius R with a twisted boundary condition, whose partition function is

$$Z_N(\Omega) = \int_{M(2\pi R)=\Omega^\dagger M(0)\Omega} \mathcal{D}M(x) \exp\left(-\operatorname{Tr}\int_0^{2\pi R} dx \left[\frac{1}{2}(\partial_x M)^2 + V(M)\right]\right), \tag{2.1}$$

where $M(x)$ is an $N \times N$ matrix-valued field on the circle and Ω is a unitary matrix. The form of $V(M)$ is

$$V(M) = \frac{1}{2}M^2 - \frac{g}{3\sqrt{N}}M^3\,, \tag{2.2}$$

but the precise form is not important in the double scaling limit. The proposed matrix model is defined by the following partition function

$$Z_N(\lambda) = \int D\Omega\; e^{\lambda \operatorname{Tr}(\Omega+\Omega^\dagger)} Z_N(\Omega)\,. \tag{2.3}$$

Note that this would be identical to the partition function of the Gross-Witten model [13] if we drop $Z_N(\Omega)$ on the right hand side. In this sense, $Z_N(\lambda)$ represents a coupled theory of Gross-Witten model and $c = 1$ matrix model.

The claim of ref. [8] is that the matrix model (2.3) describes in the double scaling limit a string theory defined on a two-dimensional black hole with temperature $T = 1/2\pi R$. This claim is based on the FZZ conjecture [7] which shows some pieces of evidence for an equivalence between $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ coset CFT with level k, which is known to be a CFT description of the two-dimensional black hole [3, 4, 5] when $k = 9/4$, and the sine-Liouville theory

$$S = \frac{1}{4\pi}\int d^2\sigma\left[(\partial x)^2 + (\partial\phi)^2 - \frac{1}{\sqrt{k-2}}R^{(2)}\phi + \lambda e^{-\sqrt{k-2}\phi}\cos\sqrt{k}(x_L - x_R)\right]. \tag{2.4}$$

See ref. [8] for complete detail. There it is shown that the matrix model (2.3) is equivalent to a modified version of the sine-Liouville theory

$$S = \frac{1}{4\pi}\int d^2\sigma\left[(\partial x)^2 + (\partial\phi)^2 - 2R^{(2)}\phi + \mu e^{-2\phi} + \lambda e^{(R-2)\phi}\cos R(x_L - x_R)\right]. \tag{2.5}$$

In a suitable limit, in which the term $\mu e^{-2\phi}$ is negligible, and with $R = 3/2$, this modified theory reduces to the sine-Liouville theory with $k = 9/4$. Evidence for this equivalence is provided by calculating the free energy of the matrix model, and the results show exact λ-dependence of the free energy expected from the worldsheet theory (2.5). Thus, by combining these two claims, it is proposed that the matrix model (2.3) with $R = 3/2$ in a suitable limit would be equivalent to a string theory whose tree level dynamics is governed by $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ coset CFT with $k = 9/4$. Note that varying R in the matrix model is not equivalent to varying k in the coset CFT. Varying R corresponds to varying the temperature, and this enables discussion of a thermodynamical issue on the black hole.

In the course of computation (2.3), the matrix M is integrated out first, and thus we cannot reduce the system to that of eigenvalues of M. This forces us to employ a method different from the ordinary matrix model in taking the double scaling limit. Instead of introducing Fermi level in canonical ensemble, ref. [8] introduced the grand canonical partition function

$$Z(\lambda,\mu) = \sum_{N=0}^{\infty} e^{2\pi R\mu N} Z_N(\Lambda)\,, \tag{2.6}$$

where the chemical potential μ takes the role of the effective Fermi level. It has been shown in [14] that the $1/\mu$ expansion of $F(\lambda,\mu) \equiv \log Z(\lambda,\mu)$ reproduces the genus expansion of $\log Z_N(\lambda)$ in the double scaling limit. The introduction of the grand canonical partition function appears to be very powerful method since $Z(\lambda,\mu)$ is the τ-function of an integrable system, and as a result, $Z(\lambda,\mu)$ can be obtained by solving a partial differential equation.

It should be noted that the matrix model should be defined with a *finite* cut-off Λ which determines the range of integration of M. In [8], it is proposed that the black hole is made of non-singlet states of $\mathrm{SU}(N)$ in the matrix model, while the non-singlet states become infinitely heavy compared with the singlet states as $\Lambda \to \infty$ [15]. Thus one should keep Λ finite as long as one would like to obtain a black hole.

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3. Black hole matrix model as a gauge theory and holography

A main reason for recent resurgence of matrix model is that the latter found a new and more appealing interpretation as an open string theory of unstable D-branes. This endowed the matrix model of $c = 1$ strings with a strong flavor of open string / closed string duality, along the line of AdS/CFT correspondence. In describing the matrix model as an open string theory, the gauging is conceptually important since, the gauge field is a universal aspect of D-brane, much as gravity is universal to critical closed string theory. In light of this, one may ask whether the correct matrix model of bosonic black hole in two dimensions should be also a gauged quantum mechanics. Here we wish to point out that the matrix model proposed by Kazakov et.al. is itself a gauge theory as it is.

For this, let us introduce $\mathrm{U}(N)$ gauge field A, again $N \times N$ matrix, and let M_0 be in the adjoint representation under this $\mathrm{U}(N)$. We elevate the original matrix model, prior to the deformation, to a gauged version by replacing the time derivative of M_0 by a covariant one

$$\partial_x M_0(x) \;\to\; \partial_x \tilde{M}(x) + i[A(x), \tilde{M}(x)]\,, \tag{3.1}$$

where we used a new notation $\tilde{M}$ to emphasize that it is now coupled to the gauge field, and at the same time integrating over the gauge field in the path integral;

$$\int \mathcal{D}M_0(x) \;\to\; \int \mathcal{D}A(x)\mathcal{D}\tilde{M}(x)\,. \tag{3.2}$$

Usually this will simply impose the Gauss constraint and do nothing else.

With periodic x, however, a new physical variable emerges from the gauge sector in the form of the holonomy, which is nothing but the untraced Wilson loop;

$$\mathcal{W} = Pe^{i\oint A\,dx}\,. \tag{3.3}$$

We may bring $A(x)$ to a constant matrix $\bar{A}$, using small gauge transformations, but only in a manner that leaves the holonomy invariant. Holonomy would change upon a non-single-valued gauge transform but the latter is not a valid gauge transformation. Thus, dividing by proper gauge volume, we find reduction of the path integral as

$$\int \mathcal{D}A(x)\mathcal{D}\tilde{M}(x) \;\to\; \int D\bar{A} \int \mathcal{D}\tilde{M}(x)\,. \tag{3.4}$$

It is important to note here that the scalar $M_0(x)$ must satisfy periodic boundary condition, for we have not modified anything other than introducing the gauge field.

Now it is also clear that we may use a large (i.e. non-single-valued) gauge transformation,

$$U = e^{ix\bar{A}}\,, \tag{3.5}$$

to induce a "change of variable," upon which we find

$$\tilde{M}(x) \quad \Rightarrow \quad \mathrm{U}(x)^\dagger \tilde{M}(x)\,\mathrm{U}(x)\,, \tag{3.6}$$

while

$$\bar{A} \quad \Rightarrow \quad 0\,. \tag{3.7}$$

Since $\tilde{M}$ is a dummy variable of the path integral, the only relevant point of this is that the transformed matrix $M' \equiv U^\dagger \tilde{M} U$ obeys a modified periodicity condition that

$$M'(2\pi R) = \mathrm{U}(2\pi R)^\dagger M'(0)\, \mathrm{U}(2\pi R)\,. \tag{3.8}$$

By now, it should be pretty clear that we must identify variables $M(x), \Omega$ of the matrix model as

$$M = M'\,, \qquad \Omega = \mathrm{U}(2\pi R)\,, \tag{3.9}$$

and

$$\int D\bar{A} \int \mathcal{D}\tilde{M}(x) = \int D\Omega \int_{M(2\pi R)=\Omega^\dagger M(0)\Omega} \mathcal{D}M(x)\,. \tag{3.10}$$

The final ingredient in making of the deformed matrix model is the insertion of

$$e^{\lambda\, \mathrm{Tr}(\Omega+\Omega^\dagger)}\,, \tag{3.11}$$

which is nothing but the exponential of the traced Wilson loop operator,

$$e^{\lambda\, \mathrm{Tr}(\mathcal{W}+\mathcal{W}^\dagger)}\,. \tag{3.12}$$

Thus we conclude that the deformed matrix model is in itself a gauged quantum mechanics. One effect of this perturbation is to modify the Gauss constraint, which otherwise would have truncated non-singlet states altogether. With finite λ, The modified Gauss constraint now allow contributions from non-singlet sector with appropriate weight determined by the Wilson loop operator.

With this modest observation, we are ready to consider a more direct comparison between the deformed matrix model and bosonic black holes in two dimensions, by making use of familiar holographic relations between gauge theories and closed string theories. Let Y denote the target spacetime of the dual string theory. Y is a two-dimensional euclidean manifold since the matrix model is supposed to describe a black hole in a thermal background. Since the matrix model is defined on S^1, Y should be asymptotically $\mathbf{R}\times S^1$. Y should also have a U(1) isometry reflecting the time-translation symmetry of the matrix model. Thus there are only two possibilities for the topology of Y; an infinitely long cylinder or an semi-infinite cigar type geometry.

With the fact that the matrix model is equivalent to a gauge theory perturbed by Wilson loop, we now have an order parameter to determine the topology of Y, and to some extent its geometry as well. Consider the expectation value of the traced Wilson loop operator defined for the time circle,

$$W \equiv \frac{1}{N}\, \mathrm{Tr}\, \mathcal{W}\,. \tag{3.13}$$

In analogy with the well-known AdS/CFT correspondence [9], $\langle W \rangle$ must be identifiable with the partition function of the dual string whose boundary is fixed on S^1, i.e.

$$\langle W \rangle = \left\langle \frac{1}{N}\, \mathrm{Tr}\, P e^{i\oint A dx} \right\rangle = \int \mathcal{D}X\; e^{-S_{NG}}\,, \tag{3.14}$$

where S_{NG} is the Nambu-Goto action and the path integral is over the worldsheets with the boundary wrapping the asymptotic circle.

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Since the dual string lives in a two-dimensional Eulidean spacetime, call it Y, it has no fluctuating degrees of freedom. This means that the path integral in the string side is evaluated by calculating the area of the string wrapping on Y, or the area of Y itself. From the observation that the unperturbed matrix model is dual to the $c = 1$ Liouville theory, it is reasonable to suppose that the shape of Y is asymptotically a cylinder to one side. As argued in [8], the matrix model is supposed to be defined with a finite cut-off, if we wish to describe the two-dimensional black hole. According to this, then, the boundary theory should be placed not at the asymptotic boundary of Y but at some finite position. In other words, when we calculate the area of the worldsheet, Y should be regarded as either a semi-infinite cylinder $\mathbf{R}_{\geq 0} \times S^1$ or a cigar with a finite length, depending on whether it corresponds to some Liouville like theory or a black hole theory.

Then the strategy to determine the topology of the spacetime is now clear. The order parameter $\langle W \rangle$ will vanish if and only if the geometry is semi-infinite cylinder,

$$\langle W \rangle = 0 \quad \Leftrightarrow \quad Y \cong \mathbf{R}_{\geq 0} \times S^1 \,, \tag{3.15}$$

while for cigar geometry, it should measure the area inside UV cut-off. We will show below that $\langle W \rangle$ has a non-vanishing value, indicating that the holographic dual string theory indeed lives on a cigar like geometry. With this, it becomes quite likely that the matrix model in question is the holographic dual of a string theory on a two-dimensional black hole.

In fact, one can also probe for more detail of the background geometry of the dual string theory by examining the value of $\langle W \rangle$. In the closed string side, this expectation value is computed by the saddle-point evaluation of string action, which contains a factor of the exponential of minus the worldsheet area. As we will see presently, this area contains a rather crude information about the black hole, yet detailed enough to distinguish between the leading α' order geometry and the α'-exact geometry. In next section, we will study this order parameter in much detail.

4. Wilson loop and α'-exact dual geometry

In this section we evaluate $\langle W \rangle$ in the matrix model, and also in its closed string dual. For the comparison, we must recall that the precise duality is proposed for $c = 26$ case of bosonic black hole, which corresponds to $k = 9/4$. In the matrix side, this is supposed to match with $R = 3/2$ [8]. Below we will see precise match of Wilson loop expectation value in terms of its leading string coupling behavior and its UV cut-off dependence.

4.1 Wilson loop from the bulk

In general, quantities derived from the matrix model may be provided as a genus expansion. The contribution of each genus would have all α' corrections since there is no expansion parameter corresponding to α'. This means that from the matrix model side, as we will show later, we obtain a result which should be correct to all orders in α'. On the other hand, the familiar black hole geometry

$$ds^2 = \alpha' k \left[dr^2 + \tanh^2 r \, d\theta^2 \right] ,$$

$$\Phi = \Phi_0 - \frac{1}{2}\log(\cosh 2r + 1)\,, \tag{4.1}$$

is derived by considering the bare lagrangian of gauged WZW model of $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ and is not exact in α'. Even though we are considering only the area of this black hole, which is a crude measure of the geometry, higher order correction could be nontrivial for the simple reason that the asymptotic circle size is of order $\sqrt{k\alpha'}$. For finite $k = 9/4$ that we are considering, a string winding around this circle has mass of order string scale, so the extended nature of the string become important.

Fortunately, an α'-exact modification of this geometry has been proposed by Dijkgraaf, Verlinde, and Verlinde via study of conformal field theory of the gauged WZW model [10]. This was further advocated by Tseytlin [11] using the following line of reasoning: The bare action of G/H gauged WZW theory may be written as

$$I_k(g, A_z, A_{\bar z}) = kI_{WZW}(h^{-1}g\bar h), -kI_{WZW}(h^{-1}\bar h) \tag{4.2}$$

in terms of usual WZW action I_{WZW}. Here, $A_z = h\partial h^{-1}$ and $A_{\bar z} = \bar h\bar\partial\bar h^{-1}$. It was then argued that the only correction is to shift the level k in each term giving us,

$$I_k^{\mathrm{exact}}(g, A_z, A_{\bar z}) = \left(k + \frac{c_G}{2}\right) I_{WZW}(h^{-1}g\bar h) - \left(k + \frac{c_H}{2}\right) I_{WZW}(h^{-1}\bar h)\,, \tag{4.3}$$

with the dual Coxeter number c_G and c_H of the respective gauge groups. One may extract the background by integrating out the gauge field as usual, and the resulting exact geometry is[2]

$$\begin{aligned} ds^2 &= \alpha'(k-2)\left[dr^2 + \frac{\tanh^2 r}{1-(2/k)\tanh^2 r}d\theta^2\right], \\ \Phi &= \Phi_0 - \frac{1}{4}\log(\cosh 2r + 1) - \frac{1}{4}\log\left(\cosh 2r + 1 + \frac{4}{k-2}\right). \end{aligned} \tag{4.4}$$

Note that the corrections to the dilaton and to the radius of the θ-direction is exponentially small in the asymptotic region, while the correction to the radial coordinate is more prominent due to $k \to (k-2)$ in the overall factor. This means among other things, the area bound by a particular UV value of Φ is different before and after the α' correction.

The value of the Nambu-Goto action is simply the area divided by $2\pi\alpha'$. The area $A(r_0)$ for a region $r \le r_0$ is

$$\begin{aligned} A(r_0) &= 2\pi\alpha'\sqrt{k(k-2)}\left[\log\left(\cosh r_0 + \sqrt{\cosh^2 r_0 + \frac{2}{k-2}}\right) - \log\left(1 + \sqrt{1 + \frac{2}{k-2}}\right)\right] \\ &= 2\pi\alpha'\sqrt{k(k-2)}r_0 + O(1). \end{aligned} \tag{4.5}$$

For large r_0, dilaton Φ behaves as

$$\Phi(r_0) - \Phi(0) = -r_0 + O(1)\,. \tag{4.6}$$

[2]One unusual aspect of this α'-corrected black hole geometry is that it has no curvature singularity when continued to the minkowskian signature [16]. The minkowskian geometry does have event horizon but no curvature singularity inside.

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Thus we conclude that

$$e^{-A/(2\pi\alpha')} \sim \exp\left(-\sqrt{k(k-2)}(\Phi(0)-\Phi(r_0))\right). \tag{4.7}$$

Note that the leading α' geometry [3, 4, 5] would have produced $\sqrt{k^2}$ in place of $\sqrt{k(k-2)}$. With $k = 9/4$, the two differs by a factor of 3, which means that the equivalence between the matrix model and string theory on the two-dimensional black hole can be tested to all orders of α' by examining the coefficient.

To summarize, we evaluated the Wilson loop expectation value of the open string side in terms of its proposed dual in the closed string side and found

$$\langle W \rangle \sim \exp\left(-\sqrt{k(k-2)}(\Phi(0)-\Phi(r_0))\right), \tag{4.8}$$

where $\Phi(r_0)$ is the value of dilaton where the Wilson line, or the boundary of the fundamental string is located while $g_{st} \equiv \exp(\Phi(0))$ is the value of string coupling at the horizon. Presently we will see that with $k = 9/4$ for which the dual pair is proposed, the coefficient $\sqrt{k(k-2)} = 3/4$ matches the result of the matrix model computation exactly.

4.2 Wilson loop from the matrix model

Now we perform the matrix model calculation. As has been shown previously, the twist matrix Ω is a holonomy along the time circle. Thus the Wilson loop operator is

$$W = \frac{1}{N}\,\mathrm{Tr}\,\mathcal{W} = \frac{1}{N}\,\mathrm{Tr}\,\Omega\,. \tag{4.9}$$

Employing the grand canonical method as in [8], we will compute,

$$\langle W \rangle \equiv \frac{1}{Z(\lambda,\mu)} \sum_{N=0}^{\infty} e^{2\pi R\mu N} \int \mathcal{D}\Omega\mathcal{D}M\; e^{\lambda\,\mathrm{Tr}(\Omega+\Omega^\dagger)-S(M)}\, \frac{1}{N}\,\mathrm{Tr}\,\Omega\,, \tag{4.10}$$

where $Z(\lambda, \mu)$ is the grand canonical partition function (2.6).

It is easy to show that $\langle W \rangle$ is a solution of the following differential equation

$$\partial_\mu \langle W \rangle + \partial_\mu \log Z(\lambda,\mu) \cdot \langle W \rangle = \pi R \partial_\lambda \log Z(\lambda,\mu)\,. \tag{4.11}$$

In the limit $\lambda \to \infty$ while μ fixed, which is the proposed limit for the matrix model to be equivalent to the two-dimensional black hole, $F(\lambda, \mu) = \log Z(\lambda, \mu)$ behaves as

$$F(\lambda,\mu) = -A\lambda^{4/(2-R)} - B\mu\lambda^{2/(2-R)} + O(\lambda^0)\,, \tag{4.12}$$

where A, B depend only on R. In ref. [8], the sign of B here appears to be ambiguous in the computation. The free energy is obtained not by explicit path integral, but rather by solving a second order differential equation which is even with respect to μ. We will fix this sign so that the expectation value of N comes out to be positive, which is sensible in the spirit of original matrix model proposal. This sign choice is on par with that of ref. [17].

In this limit, the equation (4.11) becomes

$$\partial_\mu \langle W \rangle - B\lambda^{2/(2-R)} \langle W \rangle = -\frac{4\pi R A}{2-R}\,\lambda^{(2+R)/(2-R)}\,. \tag{4.13}$$

If we ignore the derivative term, to which we will come back below, this equation has a solution

$$\langle W \rangle \sim \lambda^{R/(2-R)} . \tag{4.14}$$

Since $\lambda^{-2/(2-R)} \sim g_{st}$ [8], we finally obtain

$$\langle W \rangle \sim \exp\left(-\frac{R}{2}\log g_{st}\right) . \tag{4.15}$$

The above calculation is a bit naive, since this expression can be arbitrary large for large λ while $|\langle W \rangle| \leq 1$ since the operator inside is a sum over N complex numbers of unit modulus, divided by N. This would be due to the fact that only terms independent of the cut-off Λ are derived. Thus the problem would be solved by taking into account the cut-off Λ dependence of the matrix model explicitly. To see this, let us consider the worldsheet theory

$$S = \int d^2\sigma \frac{1}{4\pi}\left[(\partial x)^2 + (\partial\phi)^2 - 2R^{(2)}\phi + \mu e^{-2\phi} + \lambda e^{(R-2)\phi}\cos R(x_L - x_R)\right] , \tag{4.16}$$

which is equivalent to the deformed matrix model in question. The cut-off Λ is related to the maximum value ϕ_{UV} of the Liouville field ϕ. Thus the Λ-dependence can be included in μ, λ by the scaling

$$\mu \to e^{-2\phi_{UV}}\mu \,, \qquad \lambda \to e^{(R-2)\phi_{UV}}\lambda \,. \tag{4.17}$$

Note that $\mu\lambda^{-2/(2-R)}$ is invariant under this scaling. Note also that the free energy is also rescaled, but it is not relevant for calculations below. This scaling results in the scaling of the Wilson loop

$$\langle W \rangle \sim e^{-R\phi_{UV}} \exp\left(-\frac{R}{2}\log g_{st}\right) . \tag{4.18}$$

Note that this multiplicative factor is relevant since the differential equation (4.11) is inhomogeneous and it should be determined irrespective of a boundary condition for the solution. In the asymptotic region, the dilaton Φ behaves as $\Phi \sim -2\phi$. Thus we obtain

$$\langle W \rangle \sim \exp\left(-\frac{R}{2}(\Phi - \Phi_{UV})\right) . \tag{4.19}$$

This result (4.19) indicates that, according to the holographic interpretation discussed previously, the target spacetime Y has a cigar like geometry since it is non-vanishing. Moreover, since the area A read off from $\langle W \rangle$ is

$$\frac{A}{2\pi\alpha'} \sim \frac{R}{2}(\Phi - \Phi_{UV}) , \tag{4.20}$$

one can see that there is a linear dilaton background in the asymptotic region. With $R = 3/2$, this matches the closed string result above precisely since the corresponding value of k is such that the coefficient of the exponent is $3/4$, upon identifying $\Phi = \Phi(0)$ and $\Phi_{UV} = \Phi(r_0)$.

Curiously, solutions of the differential equation (4.11) can have a term

$$\exp\left(-2\pi R\int^{\mu} d\mu'\langle N\rangle\right), \tag{4.21}$$

which is the homogeneous solution of eq. (4.11). We have used the fact that $\partial_\mu F(\lambda,\mu) = 2\pi R\langle N\rangle$. Since $\langle N\rangle \sim g_{st}^{-1}$, this term looks like a non-perturbative correction to the Wilson loop. However, it is not clear to us whether this piece is physical or an artifact of taking the grand canonical ensemble. The term arises out of correlation between $1/N$ and $\mathrm{Tr}\,\Omega$, and would be absent if we computed in the canonical ensemble with a fixed large-N.

Another fine detail overlooked above is that this matrix model side computation gives the negative sign for $\langle W\rangle$ with the current sign choice such that $\langle N\rangle > 0$, unlike the bulk computation. However, Ω acts the matrix M in the adjoint representation so that its sign cannot be fixed unambiguously. We believe the sign ambiguity originates here and should be chosen as necessary.

5. Discussion

We have investigated the matrix model of ref. [8], which is proposed to be dual to sine-Liouville theory and, via FZZ conjecture, thus also dual to bosonic string theory in the two dimensional black hole background. We started with the observation that this matrix model is actually the gauged version of usual matrix model for two dimensional noncritical string theory, with a crucial deformation introduced by insertion of exponentiated Wilson loop for the time circle.

Then we argued that expectation value of the Wilson loop $\langle W\rangle$ is an order parameter that determines the topology of the euclidean spacetime of the dual string theory. This is based on usual relationship in open/closed duality of AdS/CFT type situation, where Wilson line of the boundary theory is represented by macroscopic open string with boundary at asymptotic region. In fact, $\langle W\rangle$ possesses more information than the topology, since it measures the area bound by the Wilson loop.

We showed that $\langle W\rangle$ does not vanish, confirming the cigar-like geometry of the black hole background. Furthermore we found that the Wilson loop computation reproduces exactly the right area dependency expected from the bulk side computation using Nambu-Goto action, confirming in part the conjecture that this matrix model is dual to string theory in black hole background. In comparing the two sides, a crucial ingredient is the α'-exact black hole background previously given in ref. [10, 11].

A salient point of our result is that we can extract some *geometric* data on the bulk side from the matrix model, which is in general very difficult task to realize. This is due to the simplicity of two dimensional theories. Another unusual aspect of the current computation is that it is sensitive to α' correction to all orders, since the matrix theory computation should correspond to the leading order result in g_{st} but the exact result in α'. In a sense, this is also a consistency check for the α'-exact black hole background, proposed some dozen years ago.

Let us recall that ref. [8] argues the equivalence between the two sides, mainly by computing the partition function of the matrix model and comparing it to that of sine-Liouville theory. Equivalence to the black hole theory is then via another conjecture by FZZ linking the sine-Liouville theory and the black hole theory. In the present paper, we tried to give more direct connection between the matrix model and black hole. While we borrowed heavily from computations in [8], we do not rely on the FZZ conjecture at all. Rather we made use of matrix theory partition function to map observables in the matrix model to those in string theory in the black hole background. In this sense, our computation can be taken as a strong supporting evidence for the FZZ conjecture, when taken together with main results of ref. [8].

Let us close with some open questions. It is sometimes argued that the density of states of the two-dimensional black hole exhibits the Hagedorn behavior. This is simply based on the special form of Bekenstein-Hawking entropy which has the form the black hole mass divided by the constant Hawking temperature of the black hole. The latter is then referred to as Hagedorn temperature of the system. On the other hand, it is well-known that the singlet sector of the $c = 1$ matrix model has only one (continuous) degree of freedom, namely the tachyon. It has been proposed that non-singlet states would contribute to the Hagedorn behavior.

In our view point, the matrix model in question is actually a gauge theory. Thus its physical Hilbert space has to be constrained by the Gauss constraint. While the deformation introduced by the Wilson loop insertion modifies the Gauss constraint nonlinearly, allowing contribution from non-singlet sector as well as from singlet sectors, it is unclear whether such operation can change the net number of physical states. This seems to suggest that the real physical degrees of freedom of the matrix model can be much smaller than those expected based on Hagedorn behavior of the black hole geometry. It is important to clarify this issue. Possibly one relevant fact to consider is that the deformation in question involves inserting infinite number of Wilson loops, or in other word, inserting infinite number of quark-antiquark pairs. This is because we insert an exponentiated Wilson loop.

Another interesting problem on the black hole side is the matter of Hawking radiation. The matrix model is purported to be a nonperturbative formulation of the two-dimensional string theory, and thus must contain full answer to the old questions on quantum evoluton of black holes. While we gained a lot of understanding about black holes during the last decade, those were largely for extremal and near extremal black hole with a BPS-like endpoint. For instance, the question of how to describe a large uncharged black hole is still an open question. The two-dimensional black hole is probably the simplest system where black hole must radiate away completely and be replaced by something else. We hope that our holographic view point would be helpful for understanding this fundamental problem.

Recently, more varieties of matrix models are proposed for fermionic string theories in two dimensions. The matrix theory for type 0B theory is identified in [18, 19] as the same old matrix model but with different Fermi sea configurations. The Fermi sea corresponding to type 0B string is stable even non-perturbatively, contrary to the bosonic case. Matrix theory dual of type 0A is also proposed in the same work. Also gauged version of Marinari-Parisi model [20] has been proposed to be dual to $\mathcal{N} = 2$ super Liouville theory [21].

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It goes without saying that much is needed to be clarified how the bosonic case described here elevates to fermionic cases. In particular, the case of type 0 must be addressed in the present matrix theory context, since it simply corresponds to a different choice of vacuum. Type-II case is also very intriguing, given the known mirror symmetry that asserts that $\mathcal{N}=2$ Liouville theory is identical to $\mathcal{N}=2$ $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset theory. This is rather different from the bosonic case where black hole theory is reached by a perturbation away from $c=1$ Liouville theory instead of being equivalent to it.

While this manuscript was in preparation, there appeared several papers which studied black holes in type 0 theory [22]–[25]. Authors of ref. [24] in particular mention possible role of Wilson line in formation of nonextremal black holes in their system, which we suspect to be related to the role of Wilson line in our bosonic case.

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